Acoustic Receptivity Measurements Using Modal Decomposition of a Modified Orr–Sommerfeld Equation

Jason A. Monschke, Matthew S. Kuester, & Edward B. White

Department of Aerospace Engineering
Texas A&M University

51st AIAA Aerospace Sciences Meeting, Grapevine, Texas
January 9th, 2013

With many thanks to:
Dr. William Saric, Dr. Helen Reed, Rob Downs, Bobby Ehrmann, & Ben Wilcox

Sponsored by the National Center for Hypersonic Laminar-Turbulent Transition Research
Supported by NASA and AFOSR through AFOSR grant FA9550-09-1-0341.
• Overall goal: understand and predict boundary-layer transition

• Receptivity sets the initial condition in the transition roadmap
  - Environmental disturbances become entrained in the boundary layer
  - Disturbances grow and lead to transition

• Measure receptivity by inserting known disturbances over a low baseline disturbance environment and measure the boundary-layer disturbances

*Figure: Morkovin (1994)*
Acoustic Receptivity

- Receptivity to sound is important in 2–D boundary layers
- Interaction of sound with surface features generates Tollmien–Schlichting (T–S) waves
  - Leading-Edge — Goldstein (1983)
  - Localized and Non-Localized Roughness — Crouch (1992)
  - Strong Streamwise Gradients — Kerschen (1990)

Acoustic Waves  Receptivity Region

Branch I  Branch II
Previous Acoustic Receptivity Experiments

• Experiments at ASU in the ASU Unsteady Wind Tunnel studied leading-edge receptivity to sound

• Two main experimental techniques:
  – Continuous forcing with separation of scales in complex plane
  – Pulsed forcing with conditional sampling

• Experiments were biased by:
  – Upstream traveling acoustic reflections
  – Poor frequency resolution

Saric et al. (1995)
• Acoustic wavelength is orders of magnitude larger than T–S wavelength

• Stokes wave phase nearly constant over one T–S wavelength

• Hotwire data taken at several streamwise locations

• In complex plane, spiral radius is T–S amplitude

• Drawbacks:
  − Tedious
  − Receptivity shows frequency selection
Pulsed Sound — Saric & White ('98), White et al. ('00)

- Utilizes differences in group velocities
- Pulses of 4–5 cycles
- Stokes wave passes hotwire
- After delay, T–S wave passes
- Problems:
  - Poor frequency resolution
  - Sound reflections contaminate signal
  - Time consuming
  - Inconsistent results

White et al. (2000)
• Experimental pulsed-sound results are inconsistent

• Saric & White (1998)
  – $K_I = 0.05$, $U_\infty = 8$ m/s

• White et al. (2000)
  – $K_I = 0.015$, $U_\infty = 12$ m/s
  – $K_I = 0.03$, $U_\infty = 15$ m/s

• Unclear if there is trend with freestream velocity due to limited results
Objectives

- **Goal:** Accurate acoustic receptivity measurements using continuous forcing

- **Challenges:**
  - Must reduce upstream-traveling sound (Active Noise Control)
  - Must develop Stokes/T–S wave separation method that is more time efficient (Modal Decomposition)

- Modal decomposition will require Stokes waves to be cast as solutions of a modified Orr–Sommerfeld equation
Modified Orr–Sommerfeld Equation

- Traditional Orr–Sommerfeld equations are modified to include dilation effects
  - Incompressible basic state
  - Density perturbations are allowed
  - Freestream boundary conditions relaxed

Modified Orr–Sommerfeld (O–S) Equation

\[ \mathcal{A} \frac{\partial \vec{\phi}}{\partial \eta} = \mathcal{B} \vec{\phi} \]

\[ \vec{\phi} = \left[ \hat{u}, \frac{\partial \hat{u}}{\partial \eta}, \hat{v}, \hat{p}, \frac{\partial \hat{u}}{\partial x}, \frac{\partial \hat{v}}{\partial x} \right]^T \]

\[ \phi_1|_0 = 0 \quad \phi_3|_0 = 0 \quad \phi_2|_{\infty} = 0 \quad \frac{\partial \phi_3}{\partial \eta} \bigg|_{\infty} = 0 \]
Stokes/T–S Wave Solutions

- The modified O–S equation has three solutions that are of interest to us: downstream traveling Stokes wave, upstream traveling Stokes wave, and a T–S wave.
In an experiment, all three solutions plus sting vibrations and other noise are measured. To measure T–S wave amplitudes from this contaminated signal, a biorthogonality condition is found following methods of Tumin (2003).

\[
\langle \vec{\phi}_\alpha, \vec{\psi}_{\alpha'} \rangle \bigg|_0^\infty = -i \int_0^\infty \left[ \frac{\partial \vec{\psi}_{\alpha'}^T}{\partial \eta} \frac{\partial A}{\partial \alpha} \right. \\
+ \vec{\psi}_{\alpha'}^T \left( \frac{\partial B}{\partial \alpha} + \frac{\partial^2 A}{\partial \eta \partial \alpha} \right) \left. \right] \vec{\phi}_\alpha d\eta \\
= Q_\alpha \delta(\alpha' - \alpha)
\]
Biorthogonal Decomposition

Experimental Decomposition Algorithm

1. Use the biorthogonality condition to find downstream traveling Stokes wave amplitude

2. Subtract this downstream traveling Stokes wave from experimental signal

3. Use the biorthogonality condition to find the T–S wave amplitude

- This method works well for:
  - Wide range of experimental parameters ($R$, $F$, & $M_\infty$)
  - When only streamwise velocity is known
  - Signal containing noise
Experimental Facility

• Klebanoff–Saric Wind Tunnel at Texas A&M — Hunt et al. (2010)
  – low disturbance, low speed wind tunnel
  – specifically designed for boundary-layer stability and transition work
• Acoustic treatments chosen to reduce noise in T–S passband
  – Broadband acoustic panels and acoustic foam cover portions of tunnel walls
• Sound is introduced through five McCauley subwoofers mounted upstream of the test section
  – Near-planar sound interacts with the model to create T–S waves
  – Sound level is limited to 100 dB to prevent nonlinear effects
Flat-Plate Model

- Aluminum-nickel alloy flat plate with a 20:1 modified-super-ellipse (MSE) leading edge, Lin et al. (1992)
  - Designed to eliminate surface curvature discontinuities (possible receptivity site)
  - Leading edge has been machined directly onto plate to ensure smooth interface
- Mounted vertically and off-center (56:44) in test section
- Aligned to $H = 2.59 \pm 0.005$ to ensure $\partial P / \partial x = 0$
- Trailing-edge flap ensures symmetric flow about leading edge
- Velcro strip on non-test side fixes transition location
• Closed loop control
  – Two secondary subwoofers actively cancel the diffuser reflection

• Very effective
  – Upstream-traveling acoustic waves are reduced by 20 dB
  – No effect on downstream-traveling sound

**Graph:**
- **SPL, dB re. 20 μPa**
- **Forcing Frequency, Hz**
- **Solid lines are baseline case**
- **Dashed lines are control case**
- **Black is downstream-traveling**
- **Blue is upstream-traveling**
- **Blue is upstream-traveling**
- **Dashed lines are control case**
- **Solid lines are baseline case**
Experimental Parameters

Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_b$</td>
<td>2430</td>
</tr>
<tr>
<td>$F$</td>
<td>$(56-110) \times 10^{-6}$</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>7.7 m/s</td>
</tr>
<tr>
<td>$R'$</td>
<td>$510 \times 10^3$ m$^{-1}$</td>
</tr>
<tr>
<td>$x_{vle}$</td>
<td>$55 \pm 9$ mm</td>
</tr>
</tbody>
</table>

- $R_b$ chosen to match DNS by Wanderley & Corke (2001)
Receptivity Measurement Technique

• Wall-normal hotwire scans taken at branch II

• Least-squares fit is applied to time-varying data at forcing frequency
  — Avoids frequency bins associated with Fourier Transform
  — Resulting $\hat{u}$ is then phase-locked with function generator

• By convention, the receptivity coefficient is defined to be at the branch I point

• Signal-to-noise ratio is maximized by performing decomposition at the branch II point and scaling to branch I using linear theory

Receptivity Coefficient

$$K_I = \frac{(|\hat{u}|_{TS})_{Branch\ I}}{(|\hat{u}|_{St})_{LE}} = \frac{(|\hat{u}|_{TS})_{Branch\ II}}{e^N (|\hat{u}|_{St})_{LE}}$$
Linear Growth Verification

- Receptivity coefficient is found by dividing by the growth
- Growth of T–S waves must match linear stability theory
  - Plate must be accurately aligned to ensure $\partial P/\partial x = 0$
- Top: Growth of T–S waves with tape at the branch I location
- Bottom: Growth of T–S waves with no tape
Sample Decomposition

- Decomposition results for $F = 93.2 \times 10^{-6}$
  - Frequency of maximum acoustic receptivity
  - Typical noise level

- Biorthogonal decomposition method accurately extracts Stokes/T–S wave amplitude and phase
Unlike Saric et al. (1995), no frequency selection present.

Maximum amplitude approx. twice that of the DNS results of Wanderley & Corke (2001).

Maximum occurs at same nondimensional frequency.
Conclusions

- Previous experiments had issues with frequency selection, poor frequency resolution, inconsistent results, and time consuming measurements

- Current experimental method resolves these problems
  - No frequency selection effect present in receptivity curve
  - Biorthogonal decomposition coupled with active noise control enables continuous forcing
  - Repeatable receptivity measurements
  - Measured receptivity curve differs with DNS in amplitude but matches the frequency of maximum receptivity
  - 1 boundary layer scan → 1 receptivity coefficient (3 minutes vs. several hours)
Future Work

• Biorthogonal decomposition is a powerful method to separate modes of the Orr–Sommerfeld equations
  – Used by Tumin et al. (2007) for hypersonic DNS studies
  – Also has potential DNS acoustic receptivity applications at low-speed

• Extend modified Orr–Sommerfeld equation to non-zero pressure gradient flows

• More complex geometry
  – Parabolic leading edge
  – Models at angles-of-attack
Questions?
29 Experimental BL Profiles

- Experimental Data
- Blasius Solution

\[ \eta \]

\[ \frac{u}{U_\infty} \]